Construction

Division of A Line Segment:

In order to divide a line segment internally is a given ratio m: n, where both m and n are positive integers,

we follow the following steps:

Step of construction:

(i) Draw a line segment AB of given length by using a ruler.

(ii) Draw and ray AX making an acute angle with AB.

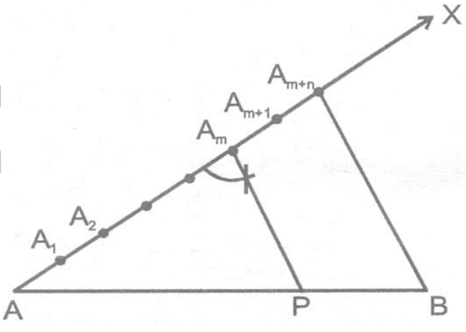
(iii) Along AX mark off (m + n) points $A_1, A_2, ..., A_{m+n}$ such that $AA_1 = A_1A_2 = ... = A_{m+n+}A_{m+n}$.

(iv) Join B A_{m+n}

(v) Through the point A_m draw a line parallel to A_{m+n} B by making an angle equal to $\angle AA_{m+n}B$ at A_m.

Suppose this line meets AB at a point P.

The point P so obtained is the required point which divides AB internally in the ratio m : n.



Example: Divide a line segment of length 12 cm internally in the ratio 3 : 2.

Sol. Following are the steps of construction.

Step of construction :

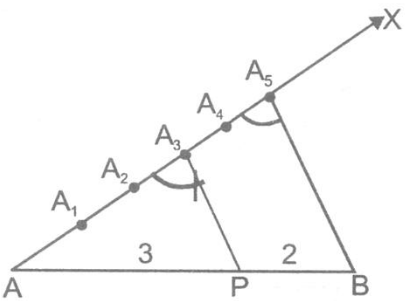
(i) Draw a line segment AB = 12 cm by using a ruler.

(ii) Draw any ray making an acute angle $\angle BAX$ with AB.

(iii) Along AX, mark-off 5 (=3 + 2) points A_1, A_2, A_3, A_4 and A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

(iv) Join BA_5

(v) Through A_3 draw a line A_3P parallel to A_5B by making an angle equal to $\angle AA_5B$ at A_3 intersecting AB at a point P.



The point P so obtained is the required point, which divides AB internally in the ratio 3 : 2.

Alternative Method For Division of A Line Segment Internally In A Given Ratio:

Use the following steps to divide a given line segment AB internally in a given ration m : n, where m and natural members.

Steps of Construction :

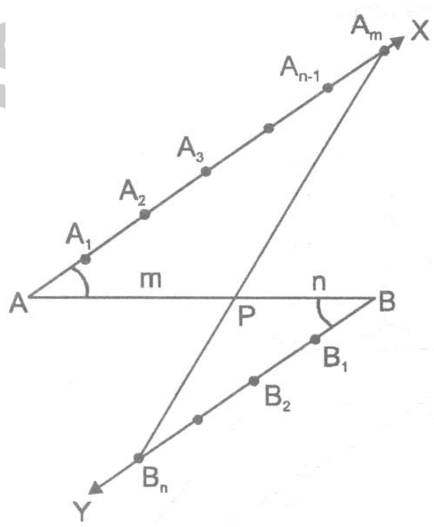
(i) Draw a line segment AB of given length.

(ii) Draw any ray AZ making an acute angle $\angle BAX$ with AB.

(iii) Draw a ray BY, on opposite side of AX, parallel to AX making an angle $\angle ABY$ equal to $\angle BAX$.

(iv) Mark off a points $A_1, A_2, ... A_{m'}$ on AX and n points $B_1, B_2, ... B_n$ on BY such that $AA_1 = A_1A_2 = ... = A_{m-1}A_m = B_1B_2 = ... B_{n-1}B_n$.

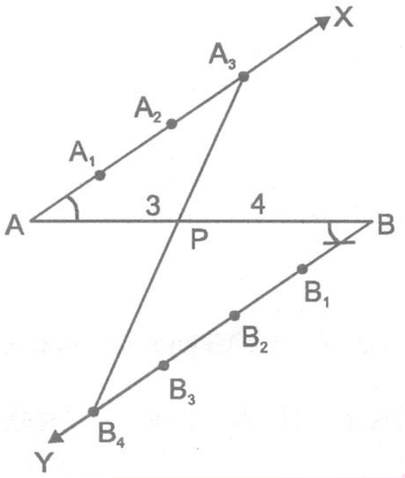
(v) Join A_mB_n. Suppose it intersect AB at P.



The point P is the required point dividing AB in the ratio m : n.

Example: Decide a line segment of length 6 cm internally in the ratio 3:4.

Sol. Follow the following steps :



Steps of Construction :

(i) Draw a line segment AB of length 6 cm.

(ii) Draw any ray AX making an acute angle $\angle BAX$ with AB.

(iii) Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.

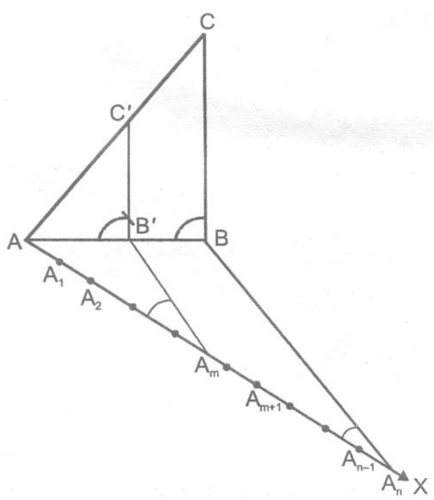
(iv) Mark of three point A_1,A_2,A_3 on AX and 4 points B_1, B_2m B_3, B_4 on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_2B_4$.

(v) Join A_3B_4. Suppose it intersects AB at a point P.

Then, P is the point dividing AB internally in the ratio 3:4.

Construction of A Triangle Similar To A Given Triangle :

Scale Factor : The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as their scale factor.



Steps of Construction when m<n :

(i) Construct the given triangle ABC by using the given data.

(ii) Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

(iii) At one end, say A, of base AB. Construct an acute angle $\angle BAX$ below the base AB.

(iv) Along AX mark of n points $A_1, A_2, A_3, \ldots A_n$ such that $AA_1 = A_1A_2 = \ldots = A_{n-1}A_n$.

(v) Join A_n B.

(vi) Draw A_mB' parallel to A_n B which meets AB at B'.

(vii) From B' draw B' C' ∥ CB meeting AC at C'.

Triangle AB'C' is the required triangle each of whose side is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of Δ ABC.

Example: Construction a Δ ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm. Now, construct a triangle similar to Δ ABC such that each of its side is two-third of the corresponding side of Δ ABC.

Sol. Steps of Construction

(i) Draw a line segment AB = 5 cm.

(ii) With A as centre and radius AC = 7 cm, draw an arc.

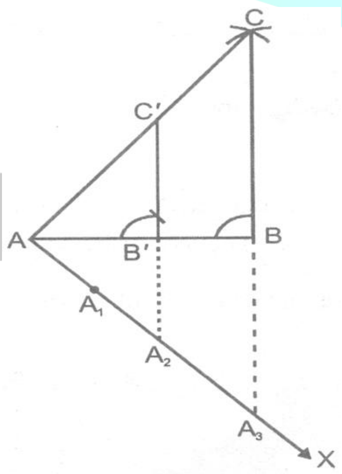
(iii) With B as centre and BC= 6 cm, draw another arc, intersecting the arc draw in step (ii) at C.

(iv) Join AC and BC to obtain Δ ABC.

(v) Below AB, make an acute angle $\angle BAX$.

(vi) Along AX, mark off three points (greater of 2 and 3 in $\frac{2}{3}$) A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$.

(vi) Join A_3B.



(viii) Draw A_2B' ∥ A_3B, meeting AB at B'.

(iv) From B', draw B'C' ∥ BC, meeting AC at C'.

AB'C' is the required triangle, each of the whose sides is two-third of the corresponding sides of Δ ABC.

Steps of Construction when m > n:

(i) Construct the given triangle by using the given data.

(ii) Take any of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

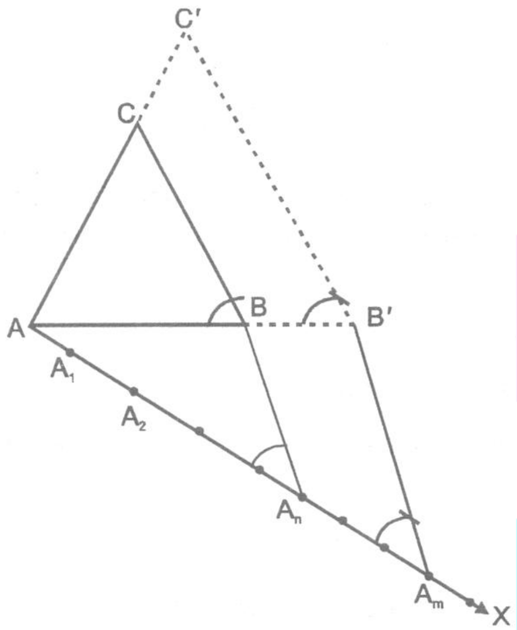
(iii) At one end, say A, of base AB construct an acute angle $\angle BAX$ below base AB i.e. on the composite side of the vertex C.

(iv) Along AX, mark-off m (large of m and n) points $A_1, A_2, ...A_m$ on AX such that $AA_1 = A_1A_2 = ... A_{m-1} A_m$.

(v) Join A_n to B and draw a line through A_m parallel to A_n B, intersecting the extended line segment AB at B'.

(vi) Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.

(vii) Δ AB'C' so obtained is the required triangle.



Example: Draw a triangle ABC with side BC = 7 cm, $\angle B = 45^0$, $\angle A = 150^0$ Construct a triangle whose side are (4/3) times the corresponding side of Δ ABC.

Sol. In order to construct Δ ABC, follow the following steps :

(i) Draw BC = 7 cm.

(ii) At B construct $\angle CBX = 45^0$ and at C construct $\angle BCY = 180^0 - (45^0 + 105^0) = 30^0$

Suppose BC and CY intersect at A. Δ ABC so obtained is the given triangle.

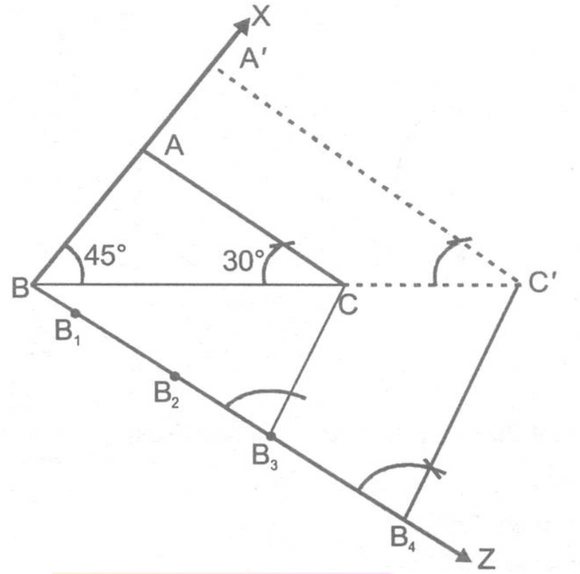
(iii) Construct an acute angle $\angle CBZ$ at B on opposite side of vertex A of Δ ABC.

(iv) Mark-off four (greater of 4 and 3 in $\frac{4}{3}$) points, B_1, B_2, B_3, B_4 on BZ such that $BB_2 - B_1B_2 = V_2B_3 = B_3B_4$.

(v) Join B_3 (the third point) to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C'.

(vi) Draw a line through C' parallel to CA intersecting the extended line segment BA at A' Triangle A'BC' so obtained is the required triangle such that

$$\frac{A'B'}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{4}{3}$$



Construction of Tangent To A Circle:

10.4 (a)To Draw the Tangent to a Circle at a Given Point on it, When the Centre of the Circle is Known :

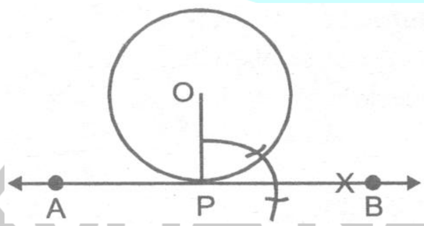
Given : A circle with centre O and a point P and it.

Required : To draw the tangent to the circle at P.

Steps of Construction.

(i) Join OP.

(ii) Draw a line AB perpendicular to OP at the point P. APB is the required tangent at P.



Example: Draw a circle of diameter 6 cm with centre O. Draw a diameter AOB. Through A or B draw tangent to the circle.

Sol. **Given :** A circle with centre O and a point P on it.

Required : To draw tangent to the circle at B or A.

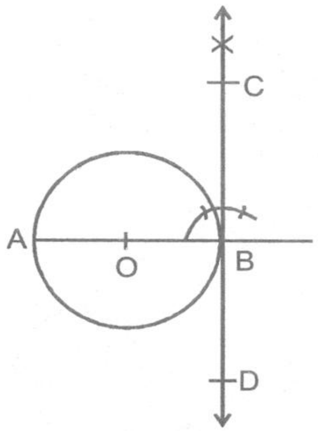
Steps of Construction.

(i) With O as centre and radius equal to 3 cm (6 $\div$ 2) draw a circle.

(ii) Draw a diameter AOB.

(iii) Draw CD $\perp$ AB.

(iv) So. CD is the required tangent.



(b) To Draw the Tangent to a Circle at a Given Point on it, When the Centre of the Circle is not Known :

Given : A circle and a point P on it.

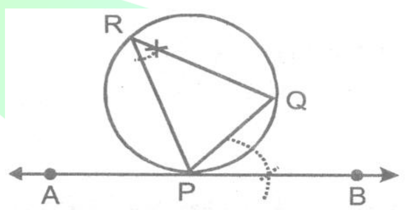
Required : To draw the tangent to the circle at P.

Steps of Construction

(i) Draw any chord PQ and Joint P and Q to a point R in major arc PQ (or minor arc PQ).

(ii) Draw $\angle QPB$ equal to $\angle PRQ$ and on opposite side of chord PQ.

The line BPA will be a tangent to the circle at P.



Example: Draw a circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction.

Sol. Given : To draw a tangent to a circle at P.

Steps of Construction

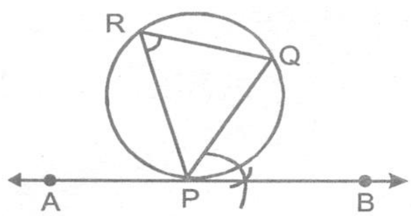
(i) Draw a circle of radius = 4.5 cm.

(ii) Draw a chord PQ, from the given point P on the circle.

(iii) Take a point R on the circle and joint PR and QR.

(iv) Draw $\angle QPB = \angle PRQ$ on the opposite side of the chord PQ.

(v) Produce BP to A. Thus, APB is the required tangent.



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(c) To Draw the Tangent to a Circle from a Point Outside it (External Point) When its Centre is known :

Given : A circle with centre O and a point P outside it.

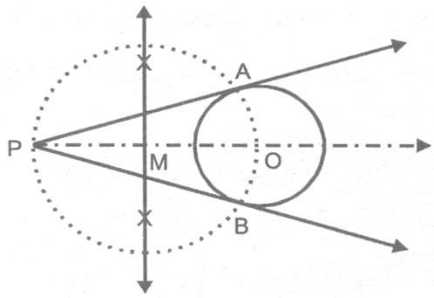
Required : To construct the tangents to the circle from P.

Steps of Construction :

(i) Join OP and bisect it. Let M be the mid point of OP.

(ii) Taking M as centre and MO as radius, draw a circle to intersect C (O, r) in two points, say A and B

(iii) Join PA and PB. These are the required tangents from P to C(O,r)



Example: Draw a circle of radius 2.5 cm. From a point P, 6 cm apart from the centre of a circle, draw two tangents to the circle.

Sol. **Given :** A point P is at a distance of 6 cm from the centre of a circle of radius 2.5 cm

Required : To draw two tangents to the circle from the given point P.

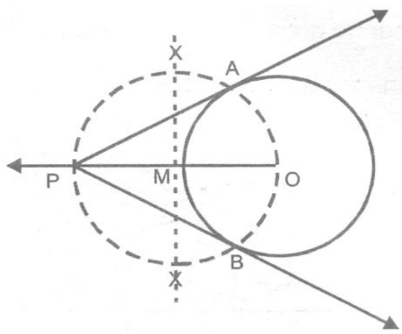
Steps of Construction :

(i) Draw a circle of radius 2.5 cm. Let it centre be O.

(ii) Join OP and bisect it. Let M be mid-point of OP.

(iii) Taking M as centre and MO as radius draw a circle to intersect C in two points, say A and B.

(iv) Join PA and PB. These are the required tangents from P to C.



(d) To Draw Tangents to a Circle From a Point Outside it (When its Centre is not Known):

Given : P is a point outside the circle.

Required : To draw tangents from a point P outside the circle.

Steps of Construction:

(i) Draw a secant PAB to intersect the circle at A and B.

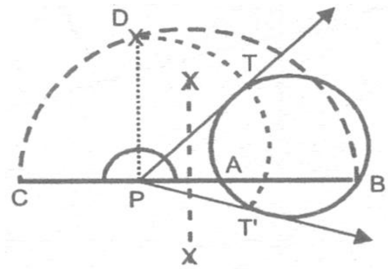
(ii) Produce AP to a point C, such that PA = PC.

(iii) With BC as a diameter, draw a semicircle.

(iv) Draw PD ⊥ CB, intersecting the semicircle at D.

(v) Taking PD as radius and P as centre, draw arcs to intersect the circle at T and T'.

(iv) Join PT and PT'. Then, PT and PT' are the required tangents.



Example: Draw a circle of radius 3 cm. From a point P, outside the circle draw two tangents to the circle without using the centre of the circle.

Given : A point P is outside the circle of radius 3 cm.

Required : To draw two tangents to the circle from the point P, without the use of centre.

Steps of constructing

(i) Draw a circle of radius 3 cm.

(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.

(iii) Produce AP to C such that AP = CP.

(iv) Draw a semicircle, wit CB as a diameter.

(v) Draw PD ⊥ AB, intersecting the semi-circle AT D.

(vi) With PD as radius and P as centre draw two arcs to intersect the given circle at T and T'.

(vii) Joint PT and PT'. Which are the required tangents.