SHIKSHA NATION

Arithmatic Progressions

Progressions:

Those sequence whose terms follow certain patterns are called progression. Generally there are three types of progression.

(i) Arithmetic Progression **(A.P.)** **(ii)** Geometric Progression **(G.P.)** (iii) Harmonic Progression **(H.P.)**

Arthmetic Progression:

A sequence is called an **A.P.,** if the difference of a term and the previous term is always same. **i.e.** $\mathbf{d = t_{n+1} - t_n =}$ **Constant** for all $n \in N$. The constant difference, generally denoted by **'d'** is called the common difference.

Example: Find the common difference of the following A.P. : 1,4,7,10,13,16 ......

Sol. 4 - 1 = 7 - 4 = 10 - 7 = 13 - 10 = 16 - 13 = 3 (constant).

$\therefore$ Common difference (d) = 3.

General Form of An A.P.:

If we denote the starting number i.e. the 1st number by **'a'** and a fixed number to the added is **'d'** then **a, a + d, a + 2d, a + 3d, a + 4d, ...** forms an **A.P.**

Example: Find the A.P. whose 1st term is 10 & common difference is 5.

Sol. Given : First term (a) = 10 & Common difference (d) = 5.

$\therefore$ A.P. is 10, 15, 20, 25, 30, ...

n^{th} Term of an A.P. :

Let A.P. be a, a + d, a + 2d, a + 3d, ...

Then, First term $(\mathbf{a_1})$ = a + 0.d

Second term $(\mathbf{a_2})$ = a + 1.d

Third term $(\mathbf{a_3})$ = a + 2.d

. .

. .

. .

n^{th} term $(\mathbf{a_n})$ = a + (n - 1) d

$\therefore$ $\mathbf{a_n = a + (n - 1)\, d}$ **is called the n^{th} term.**

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Example: Determine the A.P. whose their term is 16 and the difference of 5^{th} term from 7^{th} term is 12.

Sol. Given : $a_3 = a + (3 - 1)\,d = a + 2d = 16$ .....(i)

$a_7 - a_5 = 12$ ....(ii)

$(a + 6d) - (a + 4d) = 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Put $d = 6$ in equation (i)

$a = 16 - 12$

$a = 4$

$\therefore$ A.P. is 4, 10, 16, 22, 28, ...

Example: Which term of the sequence 72, 70, 68, 66, ... is 40 ?

Sol. Here 1^{st} term $x = 72$ and common difference $d = 70 - 72 = -2$

$\therefore$ For finding the value of n

$a_n = a + (n - 1)d$

$\Rightarrow$ $40 = 72 + (n - 1)(-2)$

$\Rightarrow$ $40 - 72 = -2n + 2$

$\Rightarrow$ $-32 = -2n + 2$

$\Rightarrow$ $-34 = -2n$

$\Rightarrow$ $n = 17$

$\therefore$ 17^{th} term is 40.

Example: Is 184, a term of the sequence 3,7,11,..... ?

Sol. Here 1^{st} term $(a) = 3$ and common difference $(d) = 7 - 3 = 4$

n^{th} term $(a_n) = a + (n - 1)\,d$

$\Rightarrow$ $184 = 3 + (n - 1)\,4$

$\Rightarrow$ $181 = 4n - 4$

$\Rightarrow$ $185 = 4n$

$\Rightarrow$ $n = \frac{185}{4}$

Since, n is not a natural number.

$\therefore$ 184 is not a term of the given sequence.

Example: Which term of the sequence 20, $19\frac{1}{2}, 18\frac{1}{2}, 17\frac{3}{4}$ is the 1st negative term.

Sol. Here 1st term (a) = 20, common difference (d) = $19\frac{1}{4} - 20 = -\frac{3}{4}$

Let nth term of the given A.P. be 1st negative term $\therefore a_n < 0$

i.e. a + (n - 1) d < 0

$\Rightarrow \quad 20 + (n - 1)\left(-\frac{3}{4}\right) < 0 \Rightarrow \frac{83}{4} - \frac{3n}{4} < 0$

$\Rightarrow \quad 3n > 83 \Rightarrow n > \frac{83}{3} \Rightarrow n > 27\frac{2}{3}$

Since, 28 is the natural number just greater then $27\frac{2}{3}$.

$\therefore$ 1st negative term is 28th.

Example: If pth, qth and rth term of an A.P. are a,b,c respectively, then show than a(q - r) + b(- p) + c(p - q) = 0.

Sol. $a_p = a \Rightarrow A + (p - 1) D = a$ ......(1)

$a_q = b \Rightarrow A + (q - 1) D = b$ .....(2)

$a_r = c \Rightarrow A + (r + 1) D = c$ ......(3)

Now, L.H.S. = a (q - r) + b(r - p) +c (p - q)

= {A + (p - 1)D} (q - r) + {A + (q - 1)D} (r - p) + {A + (r - 1)D} (p - q)

= 0. **R.H.S**

Example: If m times the mth term of an A.P. is equal to n times its nth term. Show that the (m + n)th term of the A.P.

Sol. Let A the 1st term and D be the common difference of the given A.P.

Then, $ma_m = na_n$

$\Rightarrow \quad m[A + (m - 1)D] = n[A + (n - 1)D]$

$\Rightarrow \quad A(m - 1) + D[m + n (m - n) - (m - n)] = 0$

$\Rightarrow \quad A + (m + n - 1)D = 0$

$\Rightarrow \quad a_{m+n} = 0$

Example: If the pth term of an A.P. is q and the qth term is p, prove that its nth term is (p + q - n).

Sol. $a_p = q \Rightarrow A + (p - 1) D = q$ ......(i)

& $a_q = p \Rightarrow A + (q - 1) D = p$

Solve (i) & (ii) to get D = - 1 & A = p + q - 1

$\therefore \quad a_n = A + (n - 1) D$

$a_n = (p + q - 1) + (n - 1)(-1)$

$a_n = p + q - n.$

Example: If the m^{th} term of an A.P. $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$ then show that its (mn) term is 1.

Sol. $a_m = \frac{1}{n} \Rightarrow A + (m-1)D = \frac{1}{n}$ .....(i)

& $a_m = \frac{1}{m} \Rightarrow A + (n-1)D = \frac{1}{m}$ ....(ii)

By solving (i) & (ii) $D = \frac{1}{mn}$ & $A = \frac{1}{mn}$

$\therefore \quad a_{mn} = A + (mn - 1) D = 1.$

m^{th} term of an a.p. From the end:

Let **'a'** be the 1^{st} term and **'d'** be the common difference of an **A.P.** having **n** terms. **Then m^{th}** term from the end is **$(n - m + 1)^{th}$** term from beginning or **$\{n - (m -)\}^{th}$** term from beginning.

Example: Find 20^{th} term from the end of an A.P. 3,7,11..... 407.

Sol. $407 = 3 + (n - 1)4 \Rightarrow n = 102$

$\therefore 20^{th}$ term from end $\Rightarrow m = 20$

$a_{102-(20-1)} = a_{102-19} = a_{83}$ from the beginning.

$a_{83} = 3 + (83 + 1)4 = 331.$

Selection of Terms In An A.P.:

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

No. of Terms	Terms	Common Difference
For 3 terms	a – d, a, a + d	d
For 4 terms	a – 3d, a – d, a + d, a + 3d	2d
For 5 terms	a – 2d, a – d, a, a + d, a + 2d	d
For 6 terms	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

Example: The sum of three number in A.P. is -3 and their product is 8. Find the numbers.

Sol. Three no. 's in A.P. be a - d, a, a + d

$\therefore$ $a - d + a + a + d = -3$

$3a = -3 \Rightarrow a = -1$

& $(a - d)\, a\, (a + d) = 8$

$a(a^2 - d^2) = 8$

$(-1)(1 - d^2) = 8$

$1 - d^2 = -8$

$\Rightarrow$ $d^2 = 9$

$\Rightarrow$ $d = \pm 3$

If a = 8 & d = 3 numbers are -4, -1, 2.

If a = 8 & d = - numbers are 2, -1, -4.

Sum of n terms of an a.p. :

Let A.P. be **a, a + d, a + 1d, a + 3d,...... a + (n - 1)d**

Then, $S_n = a + (a + d) + \ldots\ldots + \{a + (n - 2)d\} + \{a + (n - 1)d\}$ .....(i)

also, $S_n = \{a + (n - 1)d\} + \{a + (n - 2)d\} + \ldots\ldots + (a + d) + a$ .....(ii)

Add (i) & (ii)

$\Rightarrow$ $2S_n = 2a + (n - 1)d + 2a + (n - 1)d + \ldots\ldots\ldots\ldots + 2a + (n - 1)d$

$\Rightarrow$ $2S_n = n\,[2a + (n - 1)d]$

$\Rightarrow$ $S_n = \frac{n}{2}[2a + (n + 1)d]$

$S_n = \frac{n}{2}[a + a + (n - 1)d] = \frac{n}{2}[a + \ell]$

$\therefore$ $S_n = \frac{n}{2}[a + \ell]$ where ℓ is the last term.

Example: Find the sum of 20 terms of the A.P. 1,4,7,10.....

Sol. a = 1, d = 3

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{20} = \frac{20}{2}[2(1)+(20-1)3]$$

Example: Find the sum of all three digit natural numbers. Which are divisible by 7.

Sol. 1^{st} no. is 105 and last no. is 994.

Find n

$994 = 105 + (n + 1)7$

∴ $n = 128$

∴ Sum, $S_{128} = \frac{128}{2}[105+994]$

Properties of A.P.:

(A) For any real numbers a and b, the sequence whose n^{th} term is $\mathbf{a_n = an + b}$ is always an **A.P.** with common difference **'a'** (i.e. coefficient of term containing **n**)

(B) If any n^{th} term of sequence is a linear expression in **n** then the given sequence is an **A.P.**

(C) If a constant term is added to or subtracted from each term of an **A.P.** then the resulting sequence is also an **A.P.** with the same common difference.

(D) If each term of a given **A.P.** is multiplied or divided by a non-zero constant **K,** then the resulting sequence is also an **A.P.** with common difference **Kd** or ________ respectively. Where **d** is the common difference of the given **A.P.**

(E) In a finite **A.P.** the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of 1^{st} and **last term.**

(F) If three numbers **a,b,c** are in **A.P.,** then **2b = a + c.**

Example: Check whether $a_n = 2n^2 + 1$ is an **A.p.** or not.

Sol. $a_n = 2n^2 + 1$

Then $a_{n+1} = 2(n+1)^2 + 1$

∴ $a_{n+1} - a_n = 2(n^2 + 2n + 1) + 1 - 2n^2 - 1$

$= 2n^2 + 4n + 2 + 1 - 2n^2 - 1$

$= 4n + 2$, which is not constant

∴ The above sequence is not an A.P.